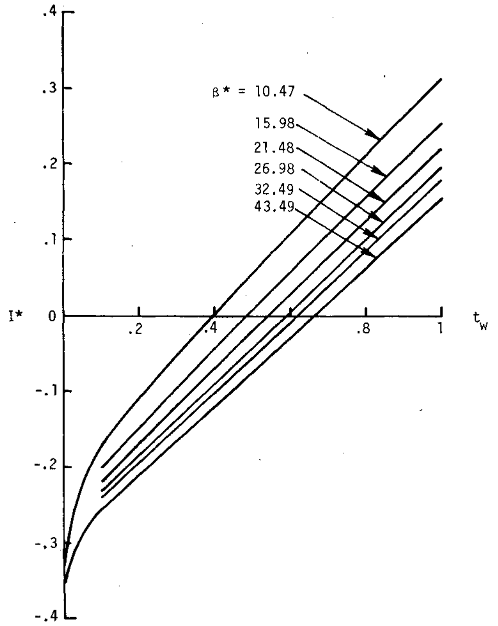


**Fig. 3** $I^*$ vs $t_w$ for $\gamma = 1.4$. The $\beta^*$ values are given by Eq. (11b), with $a = 40{,}400, \ldots$

outer expansion contributes to $(\delta^*)^*$, and produces a negative value for $(\delta^*)^*$.

## Results

In contrast to the difficult problem of numerically solving Eqs. (1) for large $\beta$,[6] it is quite easy to integrate (via Simpson's rule) Eqs. (13) to obtain $I^*$. Figure 2 shows results for $\gamma = 1.4$ and results from Ref. 6 at low $\beta^*$ (dashed curves) and at $\beta^* = \infty$. Agreement is poor for $t_w = 0$, were only the outer layer contributes to $I^*$. Inclusion of the next term in the outer expansion would improve this comparison. This figure clearly shows $I^*$ proportional to $(\beta^*)^{-1/2}$ when $t_w = 1$. Since $(\delta^*)^*$ is proportional to $(\beta^{1/2} I)^*$, Eq. (12), then $(\delta^*)^*$ itself is independent of $\beta^*$. Thus, a cold flow measurement of $(\delta^*)^*$, when $t_w = 1$, cannot be used to determine $\beta^*$.

When $0 < t_w < 1$, $I^*$ consists of a term dependent on $\gamma$ and $t_w$ and a $(\beta^*)^{-1/2}$ term, with the $(\beta^*)^{-1/2}$ term vanishing when $t_w = 0$. As is evident from Eq. (12), $(\delta^*)^*$ is positive when $I^*$ is positive. As expected, $(\delta^*)^*$ becomes negative, and $C_D$ exceeds unity, for a sufficiently large $\beta^*$ when $0 < t_w < 1$. A typical value for $\beta^*$ is 20,[3] and a hot-flow value for $(\delta^*)^*$ (say at $t_w = 0.2$) differs from its cold-flow value by the large multiplicative factor of $-1.39$, and, consequently, the hot-flow discharge coefficient exceeds unity.

Figure 3 shows the solid curves for $I^*$ replotted against $t_w$ with $\beta^*$ as the parameter. This figure suggests the following experimental procedure for determining "effective" values for $\beta^*$ and $(\delta^*)^*$. Extend the range of $t_w$ values by using a heater to preheat the He or $N_2$ normally used in cold-flow $C_D$ tests. By testing over a range of plenum temperatures, with the wall temperature at the throat more or less fixed by the water cooling, the dependence of $C_D$ on $t_w$ is established. With $\gamma$, $h^*$, and $r^*$ known, and $Re_0$ readily computed for each flow condition, Eq. (12) then provides the dependence of $(\beta^{1/2} I)^*$ on $t_w$. By comparing these experimentally derived values with corresponding theoretical ones, an effective value for $\beta^*$ is established. (This procedure produces only an "effective" value, since the theory, for example, may assume unity Prandtl number, or an estimated value for $r^*$.)

As shown by Fig. 3, $I^*$ is nearly linear with $t_w$ down to about 0.1. Thus, the foregoing $C_D$ vs $t_w$ data, when plotted as $I^*$ vs $t_w$, can be extrapolated to lower $t_w$ values (down to 0.1) to yield a $C_D$ applicable to an actual laser flow, where $t_w$ is typically 0.2 to 0.3.

## Acknowledgment

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# Incipient Separation of Leeward Flow Past a Lifting Plate in Viscous Hypersonic Flow

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## Nomenclature

$C$ = Chapman-Rubesin viscosity constant
$c$ = chord length
$M$ = Mach number
$Re$ = Reynolds number based on freestream conditions
$\alpha$ = angle of incidence
$\alpha^*$ = critical angle of incidence, Eq. (1)
$\chi$ = hypersonic viscous-inviscid interaction parameter, $(M_\infty^3 \sqrt{C}/\sqrt{Re_c})$

*Subscripts*

$\infty$ = conditions in the freestream
$c$ = based on chord length, corner position or step face, in the interaction model
exp = experimental value

## Introduction

It is well known that when a thin flat plate of finite chord is set at incidence to an oncoming supersonic/hypersonic stream, depending on the angle of incidence, flow Mach num-

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ber, and Reynolds number, the flow on the expansion side can separate. The essential mechanism of this is as follows. With the plate at an incidence to the oncoming supersonic stream, on the leeward side the Reynolds number is lower and the Mach number is higher than in the freestream. These effects create a thick boundary layer with an attendant subsonic sublayer. Further, the high pressures on the windward side propagate around the trailing edge, thereby inducing a pressure gradient which is adverse to the leeward boundary-layer flow. Also, the shock wave at the trailing edge, whose strength is proportional to the angle of incidence interacts with this boundary layer, and if the incidence is sufficiently large, the shock wave will cause the boundary layer to separate. We then have a trailing-edge stall. The flat plate at incidence is of particular interest because it provides a useful approximation to the leeward flow of hypersonic lifting surfaces in high-altitude flight. The purpose of this Note is to show that a simple correlation exists between the critical angle of incidence $\alpha^*$ at which separation would first appear and $\bar{\chi}$ which is a fundamental parameter of viscous hypersonic flow.

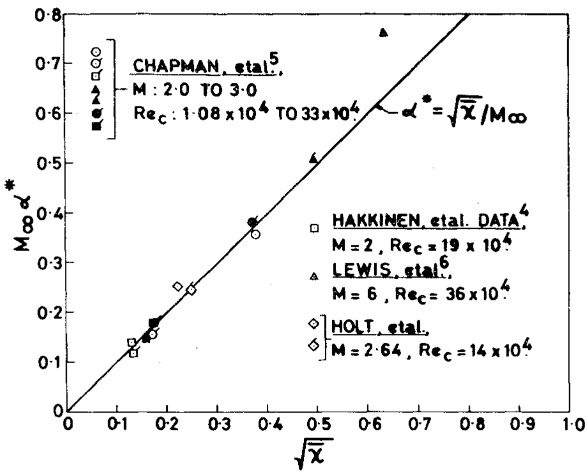


**Fig. 1 Correlation of data: $M_\infty \alpha^*$ vs hypersonic interaction parameter.**

## Analysis

Klineberg et al.[1] show in their paper that the phenomenon of separation on the expansion side is very sensitive to both Mach number and Reynolds number. Under adiabatic wall conditions the angle at which separation begins becomes smaller, the larger the Reynolds number and the smaller the Mach number. Hulcher and Behrens[2] have investigated experimentally the leeward flow of a flat plate at an angle of attack in viscous hypersonic flow. They have shown that the pressure rise in the separation interaction region is similar in form to the pressure distribution in the case of a compression corner-shock wave interaction. In particular, it was found that during the earlier part of the interaction, the pressure distribution agreed closely with the free interaction theory of Chapman. Comparison with the theoretical results of Klineberg et al.,[1] however, showed only qualitative agreement, which is not surprising since the theory does not take into account the leading-edge bluntness effects nor the influence of the pressure on the windward side propagating to the leeward side around the trailing edge.

Even though Ref. 1 discusses at length separation and its effects, no method is suggested for calculating the angle at which separation would begin. On the other hand, Hulcher and Behrens found that, for their experimental conditions, separation first occurred when the angle of attack was about 8°. However, Brown and Stewartson[3] in their discussion of the trailing-edge stall, suggest an analytical relation for this critical angle of incidence. The expression is

$$\alpha^* \sim \left[\frac{C(M_\infty^2 - 1)}{Re_c}\right]^{1/4} \tag{1}$$

Although strictly valid for a very high Reynolds number flow, it is shown here, using the available experimental data, that this relation yields reasonable estimates of the angle for the onset of separation.

Equation (1) may be simplified by letting $\sqrt{M_\infty^2 - 1} \doteq M_\infty$ (this approximation results in an error of about 13% in the pressure change at $M_\infty = 2$ and less than 5% at $M_\infty = 3$). This then results in

$$\alpha^* \sim \frac{1}{M_\infty} \cdot \left[\frac{M_\infty^2 \sqrt{C}}{\sqrt{Re_c}}\right]^{1/2} \quad \text{or} \quad \alpha^* \sim \sqrt{\bar{\chi}}/M_\infty \tag{2}$$

where $\bar{\chi}$ is the hypersonic viscous-inviscid interaction parameter.

## Discussion of Results

Equation(2) shows that a correlation should exist between $\alpha^*$ and $\sqrt{\bar{\chi}}/\Omega_\infty$. Reference 3 also suggests that the proportionality constant in Eq. (2) is of the order of unity for a plate at incidence. As a simplest assumption we consider, a priori, that the proportionality constant is unity.

It has been shown in Ref. 2 that the leeward flow separation is not unlike that of free interaction separation. It was, therefore, of interest to check the validity of Eq. (2) with the available experimental data. Figure 1 shows some of the data on laminar separation. These data provide the pressure ratio required for incipient separation. From these one could compute[8] $(\sqrt{\bar{\chi}})_{\exp}$ which, in turn, gives $(M_\infty \alpha^*)_{\exp}$ on using Eq. (2) with a proportionality constant of unity. Figure 1 is, thus, a representation of $(M_\infty \alpha^*)_{\exp}$ plotted against the true $\sqrt{\bar{\chi}}$ and it is seen, a posteriori, that the experimental data correlate very well on the basis of $M_\infty \alpha^* = \sqrt{\bar{\chi}}$. Similar correlation, valid for a cold wall compression ramp at large hypersonic Mach numbers has been shown to exist by Hankey.[9]

Based on the relation $\alpha^* = \sqrt{\bar{\chi}}/M_\infty$, the critical angle of separation, on the leeward surface for the experimental conditions of Ref. 2, turns out to be about 7°, whereas the figure quoted there is 8°. The agreement is thus reasonable. Similarly, calculations for the data of Ref. 1 indicate that separation would first occur when the plate incidence is about 4.5° at $M_\infty = 4.0$ whereas it would be 5.5° at $M_\infty = 6.0$. This again is quite reasonable when it is noted from the figures of Ref. 1 that at $M_\infty = 4.0$ and $\alpha = 10°$, separation point has already advanced about 0.7 in. upstream of the trailing edge. At $M_\infty = 6.0$ and $\alpha = 15°$, it is 1 in. upstream of the trailing edge.

In conclusion, therefore, a reasonable estimate of the critical angle for the trailing-edge stall may be obtained for the simple case of an adiabatic flat plate at incidence in viscous hypersonic flow, if the Mach number and the Reynolds number are known.

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# Direct Simulation Calculations of the Rarefied Flow Past a Forward-Facing Step

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## Nomenclature

| | |
|---|---|
| $a_\infty$ | = freestream speed of sound |
| $\epsilon$ | = molecular rotational energy |
| $P_{fm}$ | $= \frac{1}{2}P_\infty(1+\sqrt{T_w/T_\infty})$; free molecular pressure for flat-plate flow |
| $T$ | = translational temperature |
| $T'$ | = 91.5 K; potential well temperature for Morse potential |
| $T_x$ | = $x$ component of $T$ |
| $T_R$ | = rotational temperature |
| $T_w$ | = body temperature |
| $u,v$ | = average gas velocity in $x$ and $y$ directions, respectively |
| $V_x$, $V_y$, $V_z$ | = $x$, $y$, and $z$ components of molecular velocity |
| $\lambda_\infty$ | = freestream mean free path |

## Introduction

THE development of the Direct Simulation Monte Carlo method[1] for the numerical simulation of the full nonlinear Boltzmann equation has made possible the calculation of rarefied transition flow about a variety of basic body shapes. Cases treated to date using this method include axisymmetric flow about spheres[2] and two-dimensional flow past cylinders[3] and the leading edge of flat plates[4-6] aligned with the freestream. Although much of this work has been for monatomic gases, schemes for the modeling of diatomic gases are now available, one of which has been applied to the flat plate[5,6] problem by the present authors.

In the present Note, we describe a detailed Direct Simulation calculation of the two-dimensional rarefied hypersonic flow of a diatomic gas past a forward-facing step placed $48\lambda_\infty$ downstream of a flat-plate leading edge. This basic departure from the flat-plate case is interesting, since it represents the interaction of a kinetic flow in the process of transition to a merged layer with a simple geometrical obstruction, leading to the possibility of step-induced flow separation.

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The Direct Simulation method is a statistical technique in which the time evolution, including collisions, of a populous gasdynamic system is numerically simulated by following the simultaneous motion of a many orders-of-magnitude smaller number of sample particles. For a particular application, given a set of initial and boundary conditions together with a binary collision model the technique may be used to obtain statistical estimates of all desired flow properties including the distribution function, at any point in the gas. Accounts of the method and its relation to the Boltzmann equation may be found in Refs. 1 and 7.

### Binary Collision Model

The binary collision model used in the present calculation is described in detail in Ref. 6. Briefly, the collision deflection angle and the Monte Carlo conditional collision probability[1] were treated using the spherical Morse potential.[8] For the treatment of the rotational-translation energy exchange in inelastic collisions with two internal degrees of freedom in rotation, a hybrid classical/statistical technique was employed. This method essentially combines a treatment of the inelastic exchange based on classical perturbation methods for a nonspherical potential[8] with a simple statistical relaxation technique.[7] The hybrid model is constructed so as to drive the gas into a state of local energy equipartition while retaining, for pair relative translational energy $\epsilon_t > 2kT'$, far from equilibrium, a detailed microscopic description of the exchange process. The model is efficient, requiring about 0.3 sec of CDC 7600 machine time per 1000 collision calculations. It correctly simulates rotational-translational temperature relaxation with the appropriate temperature dependence of the relaxation time,[8] and leads to approximate energy equipartition at equilibrium for temperatures up to 2000 K. At equilibrium, the velocity distribution is Maxwellian but due to overall violation of detailed balancing, the Boltzmann rotational energy distribution is not achieved.

### Results and Discussion

The flow and body conditions, chosen to match those used in nitrogen experiments by Jeffrey[9] were $M_\infty = 22.9$, $T_\infty = 20$ K, $T_w/T_\infty = 14.4$. The freestream pressure and number density in these experiments were $P_\infty = 0.276$ N/m$^2$ and

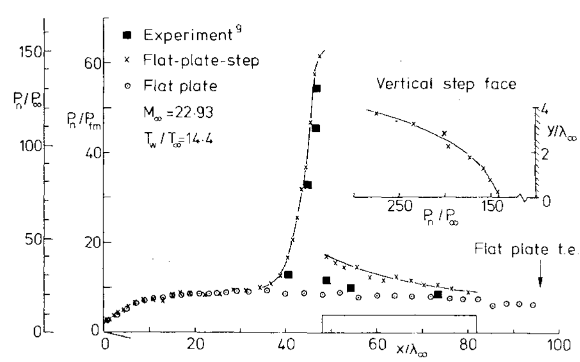


**Fig. 1 Normal surface pressure.**

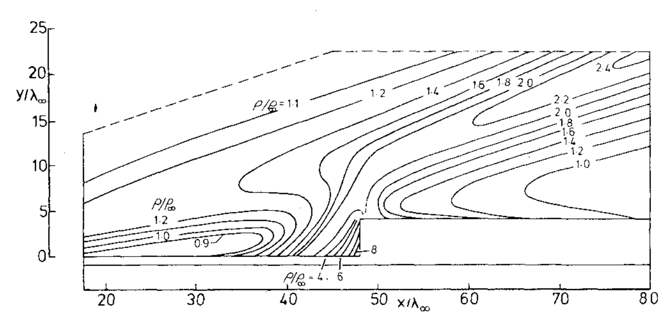


**Fig. 2 Density contours $M = 22.9$, $T_w/T_\infty = 14.4$.**